N-Gram Language Models

CMSC 473/673 - NATURAL LANGUAGE PROCESSING

Slides modified from Dr. Frank Ferraro

Grad Assignment is Released

https://laramartin.net/NLP-class/homeworks/grad.html

Learning Objectives

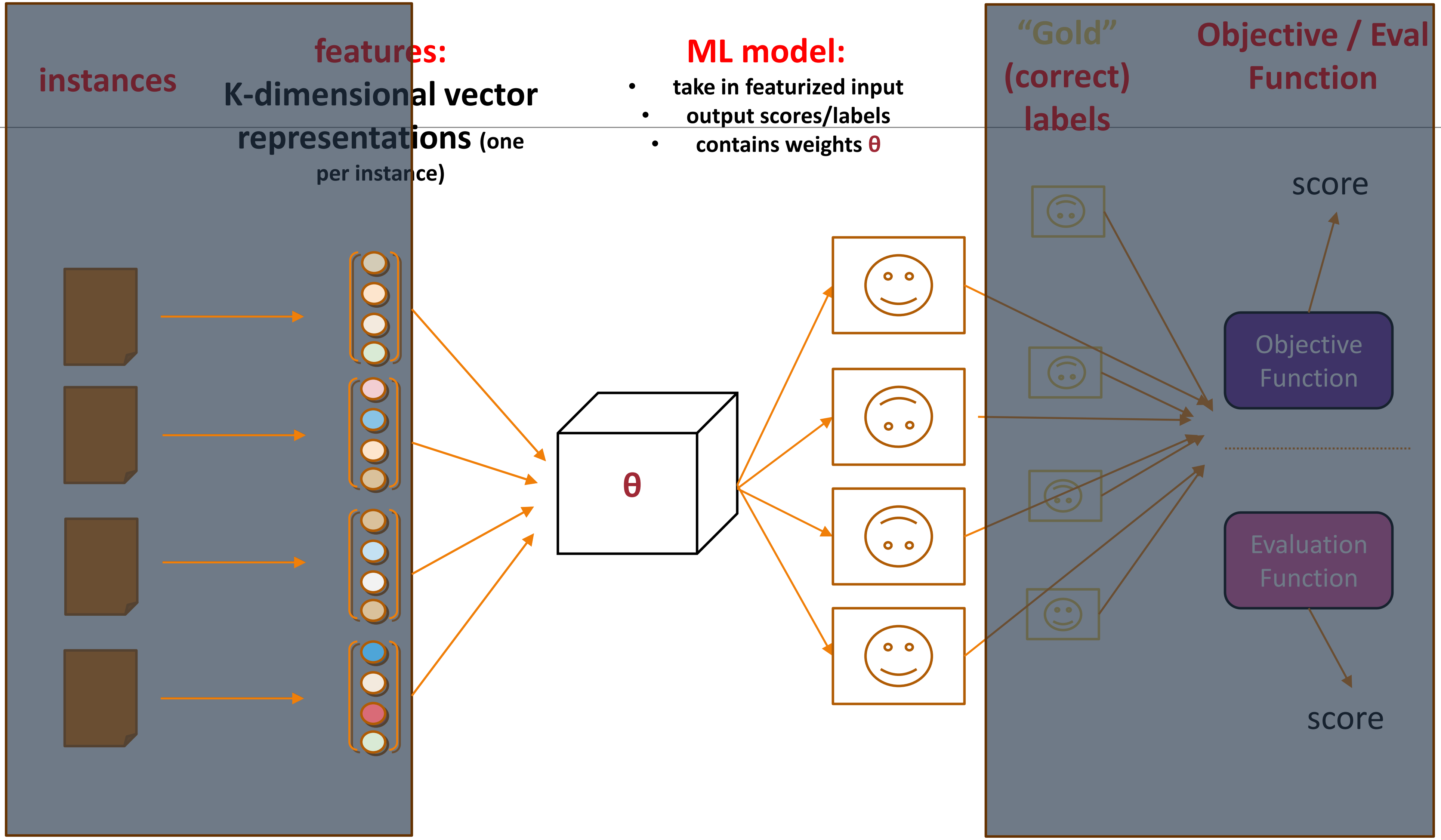
Formalize what a language model is using the Markov assumption

Code a LM using Maximum Likelihood Estimation (MLE)

Evaluate LMs with perplexity

Create a LM using smoothed counts

Defining the Model



Goal of Language Modeling

$$p_\theta(\ \textit{[...text..]}\ )$$

Learn a probabilistic model of text

Accomplished through observing text and updating model parameters to make text more likely

Two Perspectives: Prediction vs. Generation

“Prediction”

Given observed word tokens $w_1 \dots w_{N-1}$, create a classifier p to predict the next word w_N

$$p(w_N = v \mid w_1 \dots w_{N-1})$$

Two Perspectives: Prediction vs. Generation

"Prediction"

Given observed word tokens $w_1 \ldots w_{N-1}$, create a classifier p to predict the next word w_N

$$p(w_N = v \mid w_1 \ldots w_{N-1}), \text{ e.g.,}$$
$$p(w_N = \text{meowed} \mid \text{The, fluffy, cat})$$

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Given observed word tokens $w_1 \dots w_{N-1}$, create a classifier p to predict the next word w_N

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$$p(w_N = \text{meowed} \mid \text{The, fluffy, cat})$$

"Generation"

Develop a probabilistic model p to *explain/score* the word sequence $w_1 \dots w_N$

$$p(w_1 \dots w_N), \text{ e.g.,}$$
$$p(\text{The, fluffy, cat, meowed})$$

Design Question 1: What Part of Language Do We Estimate?

$$p_\theta(\ \textit{[...text..]}\ )$$

Is *[…text..]* a

- Full document?
- Sequence of sentences?
- Sequence of words?
- Sequence of characters?

A: It's task-dependent!

Design Question 2: How do we estimate robustly?

$$p_{\theta}(\textit{[...typo-text..]})$$

What if *[…text..]* has a typo?

Design Question 3: How do we generalize?

$$p_\theta(\textit{[...synonymous-text..]})$$

What if *[…text..]* has a word (or character or…) we've never seen before?

Key Idea: Probability Chain Rule

$$p(x_1, x_2, \ldots, x_S) =$$

$$p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_1, x_2) \cdots p(x_S \mid x_1, \ldots, x_{S-1})$$

Key Idea: Probability Chain Rule

$$p(x_1, x_2, \dots, x_S) =$$
$$p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_1, x_2) \cdots p(x_S \mid x_1, \dots, x_{S-1}) =$$
$$\prod_i^S \underbrace{p(x_i \mid x_1, \dots, x_{i-1})}_{\text{Language modeling is about how to estimate each of these factors in \{great, good, sufficient, …\} ways}}$$

Example: Develop a Probabilistic Email Classifier

Input: an email (all text)

Output (Gmail categories):

Primary, Social, Forums, Spam

$$\text{argmax}_{\text{y}}\, p(\text{label } Y = y \mid \text{email } X)$$

Approach #1: Discriminatively trained

Approach #2: Using Bayes rule

Classify Using Bayes Rule

$$p(\text{label } Y \mid \text{email } X) \propto p(X \mid Y) * p(Y)$$

Classify Using Bayes Rule

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Q: Why is p(Y | X) what we want to model?

Classify Using Bayes Rule

$$p(\text{label } Y \mid \text{email } X) \propto p(X \mid Y) * p(Y)$$

$$p(\text{Primary} \mid \text{Won't you please donate?}) \propto p(\text{Won't you please donate?} \mid \text{Primary})\, p(\text{Primary})$$

A Closer Look at $p(\text{Primary})$

This is the **prior probability** of each *class*

Answers the question: without knowing anything specific about a document, how likely is each class?

A Closer Look at $p($ Primary $)$

This is the **prior probability** of each *class*

Answers the question: without knowing anything specific about a document, how likely is each class?

Q: What’s an easy way to estimate it?

A Closer Look at $p($ Won't you please donate? $|$ Primary $)$

This is a *class specific* language model

A Closer Look at $p($ Won't you please donate? $|$ Primary $)$

This is a *class specific* language model

$p($ Won't you please donate? $|$ Primary $)$ is different from

$p($ Won't you please donate? $|$ Social $)$ is different from

$p($ Won't you please donate? $|$ Forums $)$ …

A Closer Look at $p($Won’t you please donate? $|$ Primary$)$

This is a *class specific* language model

To learn $p($Won’t you please donate? $|$ Class$)$:

For each class Class:

- Get a bunch of Class documents D_{Class}
- Learn a new language model p_{Class} on just D_{Class}

Language Models & Smoothing

Maximum likelihood (MLE): simple counting

Other count-based models

- Laplace smoothing, add- λ ← Easy to implement
- Interpolation models
- Discounted backoff
- Interpolated (modified) Kneser-Ney
- Good-Turing
- Witten-Bell

Advanced/ out of scope

Maxent n-gram models ← Featureful LMs

Neural n-gram models ← Feedforward LMs

Recurrent/autoregressive NNs ← Precursor to modern LMs

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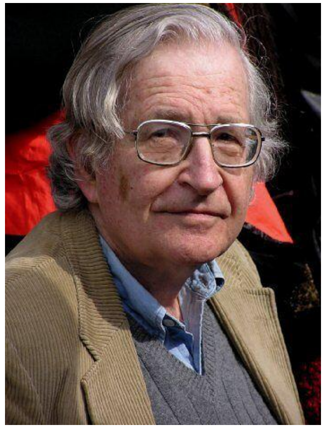
“Colorless green ideas sleep furiously”



Chomsky, Noam. Syntactic structures. Mouton & Co., 1957.

N-Grams

Maintaining an entire inventory over sentences could be too much to ask

Store “smaller” pieces?

p(Colorless green ideas sleep furiously)

N-Grams

Maintaining an entire *joint* inventory over sentences could be too much to ask

Store “smaller” pieces?

p(Colorless green ideas sleep furiously) =
p(Colorless) *

N-Grams

Maintaining an entire *joint* inventory over sentences could be too much to ask

Store “smaller” pieces?

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p(Colorless) *
p(green | Colorless) *

N-Grams

Maintaining an entire *joint* inventory over sentences could be too much to ask

Store “smaller” pieces?

p(Colorless green ideas sleep furiously) =
p(Colorless) *
p(green | Colorless) *
p(ideas | Colorless green) *
p(sleep | Colorless green ideas) *
p(furiously | Colorless green ideas sleep)

N-Grams

p(furiously | Colorless green ideas sleep)

How much does “Colorless” influence the choice of “furiously?”

N-Grams

p(furiously | Colorless green ideas sleep)

How much does “Colorless” influence the choice of “furiously?”

Remove history and contextual info

N-Grams

p(furiously | Colorless green ideas sleep)

How much does “Colorless” influence the choice of “furiously?”

Remove history and contextual info

p(furiously | Colorless green ideas sleep) ≈

p(furiously | ~~Colorless green~~ ideas sleep)

N-Grams

p(furiously | Colorless green ideas sleep)

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Remove history and contextual info

p(furiously | Colorless green ideas sleep) ≈

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N-Grams

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N-Grams

p(Colorless green ideas sleep furiously) =
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Trigrams

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Trigrams

p(Colorless green ideas sleep furiously) =
p(Colorless | *<BOS> <BOS>*) *
p(green | *<BOS>* Colorless) *
p(ideas | Colorless green) *
p(sleep | green ideas) *
p(furiously | ideas sleep)

Consistent notation: Pad the left with <BOS> (beginning of sentence) symbols

Trigrams

p(Colorless green ideas sleep furiously) =
p(Colorless | *<BOS> <BOS>*) *
p(green | *<BOS>* Colorless) *
p(ideas | Colorless green) *
p(sleep | green ideas) *
p(furiously | ideas sleep) *
p(*<EOS>* | sleep furiously)

Consistent notation: Pad the left with <BOS> (beginning of sentence) symbols
Fully proper distribution: Pad the right with a single <EOS> symbol

N-Gram Terminology

n	Commonly called	History Size (Markov order)	Example
1	unigram	0	p(furiously)

N-Gram Terminology

n	Commonly called	History Size (Markov order)	Example
1	unigram	0	p(furiously)
2	bigram	1	p(furiously | sleep)

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n	Commonly called	History Size (Markov order)	Example
1	unigram	0	p(furiously)
2	bigram	1	p(furiously | sleep)
3	trigram (3-gram)	2	p(furiously | ideas sleep)

N-Gram Terminology

n	Commonly called	History Size (Markov order)	Example
1	unigram	0	p(furiously)
2	bigram	1	p(furiously | sleep)
3	trigram (3-gram)	2	p(furiously | ideas sleep)
4	4-gram	3	p(furiously | green ideas sleep)
n	n-gram	n-1	$p(w_i \mid w_{i-n+1} \ldots w_{i-1})$

N-Gram Probability

$$p(w_1, w_2, w_3, \cdots, w_S) =$$

$$\prod_{i=1}^{S} p(w_i | w_{i-N+1}, \cdots, w_{i-1})$$

Count-Based N-Grams (Unigrams)

$$p(\text{item}) \propto count(\text{item})$$

Count-Based N-Grams (Unigrams)

$$p(\mathrm{z}) \propto count(\mathrm{z})$$

Count-Based N-Grams (Unigrams)

word type

word type

$$p(\mathrm{z}) \propto count(\mathrm{z})$$

$$= \frac{count(\mathrm{z})}{\sum_v count(\mathrm{v})}$$

word type

Count-Based N-Grams (Unigrams)

word type

word type

$$p(\mathrm{z}) \propto count(\mathrm{z})$$

$$= \frac{count(\mathrm{z})}{W}$$

number of tokens observed

Count-Based N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type) z	Raw Count count(z)	Normalization	Probability p(z)
The	1		
film	2		
got	1		
a	2		
great	1		
opening	1		
and	1		
the	1		
went	1		
on	1		
to	1		
become	1		
hit	1		
.	1		

Count-Based N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type) z	Raw Count count(z)	Normalization	Probability p(z)
The	1	16	
film	2		
got	1		
a	2		
great	1		
opening	1		
and	1		
the	1		
went	1		
on	1		
to	1		
become	1		
hit	1		
.	1		

Count-Based N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type) z	Raw Count count(z)	Normalization	Probability p(z)
The	1	16	1/16
film	2		1/8
got	1		1/16
a	2		1/8
great	1		1/16
opening	1		1/16
and	1		1/16
the	1		1/16
went	1		1/16
on	1		1/16
to	1		1/16
become	1		1/16
hit	1		1/16
.	1		1/16

Count-Based N-Grams (Trigrams)

order matters in conditioning

order matters in *count*

$$p(\mathrm{z}|\mathrm{x},\mathrm{y}) \propto count(\mathrm{x},\mathrm{y},\mathrm{z})$$

Count of the sequence of items "x y z"

Count-Based N-Grams (Trigrams)

order matters in conditioning

order matters in *count*

$$p(\mathrm{z}|\mathrm{x},\mathrm{y}) \propto count(\mathrm{x},\mathrm{y},\mathrm{z})$$

count(x, y, z) ≠ count(x, z, y) ≠ count(y, x, z) ≠ …

Count-Based N-Grams (Trigrams)

$$p(\mathrm{z}|\mathrm{x},\mathrm{y}) \propto count(\mathrm{x},\mathrm{y},\mathrm{z})$$

$$= \frac{count(\mathrm{x},\mathrm{y},\mathrm{z})}{\sum_{v} count(\mathrm{x},\mathrm{y},\mathrm{v})}$$

Count-Based N-Grams (Trigrams)

The film got a great opening and the film went on to become a hit .

Context: x y	Word (Type): z	Raw Count	Normalization	Probability p(z | x y)
The film	The	0	1	0/1
The film	film	0		0/1
The film	got	1		1/1
The film	went	0		0/1
…				
a great	great	0	1	0/1
a great	opening	1		1/1
a great	and	0		0/1
a great	the	0		0/1
…				

Count-Based N-Grams (Lowercased Trigrams)

the film got a great opening and the film went on to become a hit .

Context: x y	Word (Type): z	Raw Count	Normalization	Probability: p(z | x y)
the film	the	0	2	0/2
the film	film	0		0/2
the film	got	1		1/2
the film	went	1		1/2
…				
a great	great	0	1	0/1
a great	opening	1		1/1
a great	and	0		0/1
a great	the	0		0/1
…				

Implementation: EOS Padding

Create an end of sentence (“chunk”) token <EOS>

Don’t estimate p(<BOS> | <EOS>)

Training & Evaluation:

1. Identify “chunks” that are relevant (sentences, paragraphs, documents)
2. Append the <EOS> token to the end of the chunk
3. Train or evaluate LM as normal

Implementation: Memory Issues

Let V = vocab size, W = number of ***observed*** n-grams

Often, $W \ll V$

Dense count representation: $O(V^n)$, but many entries will be zero

Sparse count representation: $O(W)$

Sometimes selective precomputation is helpful (e.g., normalizers)

Implementation: Unknown words

Create an unknown word token <UNK>

Training:

1. Create a fixed lexicon L of size V
2. Change any word not in L to <UNK>
3. Train LM as normal

Evaluation:

Use UNK probabilities for any word not in training

A Closer Look at Count-based $p($ Won’t you please donate? $|$ Primary $)$

This is a *class specific* language model

To learn $p($ Won’t you please donate? $|$ Class $)$:

For each class Class:

- Get a bunch of Class documents D_{Class}
- Learn a new language model p_{Class} on just D_{Class}

Two Ways to Learn Class-specific Count-based Language Models

1. Create different count table(s) $c_{\text{Class}}(\ldots)$ for each Class

e.g., record separate trigram counts for Primary vs. Social vs. Forums vs. Spam

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1. Create different count table(s) $c_{\text{Class}}(\ldots)$ for each Class

    e.g., record separate trigram counts for Primary vs. Social vs. Forums vs. Spam

OR

2. Add a dimension to your existing tables $c(\text{Class}, \ldots)$

    e.g., record how often each trigram occurs within Primary vs. Social vs. Forums vs. Spam documents

Coding Knowledge Check: Make a Trigram LM

$$p(\mathrm{z}|\mathrm{x},\mathrm{y}) = \frac{count(\mathrm{x},\mathrm{y},\mathrm{z})}{\sum_{v} count(\mathrm{x},\mathrm{y},\mathrm{v})}$$

Evaluating Language Models

What is "correct?"

What is working "well?"

fine-tune any secondary (hyper)parameters

Training Data

Dev Data

Test Data

learn model parameters:
- acquire primary statistics
- learn feature weights

perform final evaluation

DO NOT TUNE ON THE TEST DATA

Evaluating Language Models

What is "correct?"

What is working "well?"

Extrinsic: Evaluate LM in downstream task

Test an MT, ASR, etc. system and see which LM does better

Issue: Propagate & conflate errors

Evaluating Language Models

What is "correct?"

What is working "well?"

Extrinsic: Evaluate LM in downstream task

Test an MT, ASR, etc. system and see which LM does better

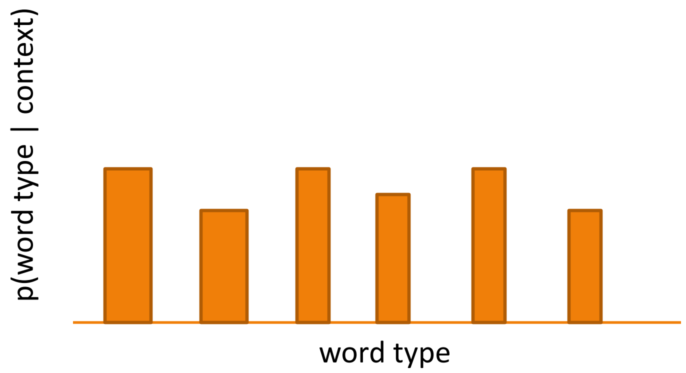
Issue: Propagate & conflate errors

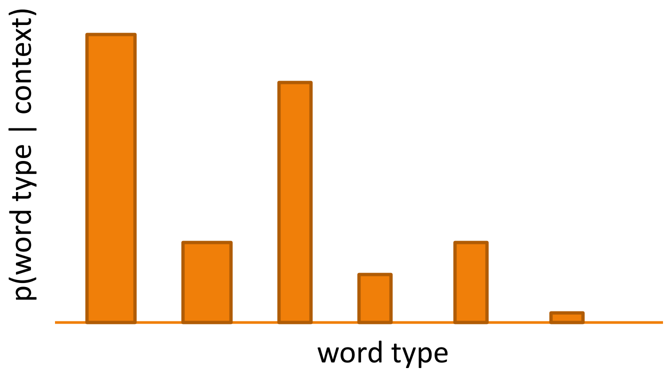
Intrinsic: Treat LM as its own downstream task

Use perplexity (from information theory)

Perplexity: Average “Surprisal”

Lower is better : lower perplexity ➔ less surprised





Less certain ➔
More surprised ➔
Higher perplexity

More certain ➔
Less surprised ➔
Lower perplexity

Perplexity

Lower is better : lower perplexity ➔ less surprised

$$\text{perplexity} = \exp(\text{avg crossentropy})$$

Perplexity

Lower is better : lower perplexity ➔ less surprised

$$\text{perplexity} = \exp\left(\frac{-1}{M}\log p(w_1, \ldots, w_M)\right)$$

Perplexity

Lower is better : lower perplexity ➔ less surprised

e.g., n-gram history
(n-1 items)

$$\text{perplexity} = \exp\left(\frac{-1}{M} \sum_{i=1}^{M} \log p(w_i \mid h_i)\right)$$

Perplexity

Lower is better : lower perplexity ➔ less surprised

$$\text{perplexity} = \exp\left(\frac{-1}{M} \sum_{i=1}^{M} \log \underbrace{p(w_i \mid h_i)}_{\geq 0,\ \leq 1:\ \text{higher}}\right)$$

Perplexity

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$$\text{perplexity} = \exp\left(\frac{-1}{M}\sum_{i=1}^{M} \overbrace{\log \underbrace{p(w_i \mid h_i)}_{\geq 0,\ \leq 1\text{: higher}}}^{\leq 0\text{: higher}}\right)$$

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Perplexity

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base must be the same

$$\text{perplexity} = \underbrace{\exp(\overbrace{\frac{-1}{M}\underbrace{\sum_{i=1}^{M}\overbrace{\log \underbrace{p(w_i \mid h_i)}_{\geq 0,\ \leq 1:\ \text{higher}}}^{\leq 0:\ \text{higher}}}_{\leq 0,\ \text{higher}}}^{\geq 0,\ \text{lower is better}})}_{\geq 0,\ \text{lower}}$$

Perplexity

Lower is better : lower perplexity ➔ less surprised

$$\text{perplexity} = \exp\left(\frac{-1}{M}\sum_{i=1}^{M} \log p(w_i \mid h_i)\right)$$

$$= \sqrt[M]{\underbrace{\prod_{i=1} \frac{1}{p(w_i \mid h_i)}}_{\text{weighted geometric average}}}$$

How to Compute Average Perplexity

If you have a list of the probabilities for each observed n-gram "token:"

```
numpy.exp(-numpy.mean(numpy.log(probs_per_trigram_token)))
```

If you have a list of observed n-gram "types" t and counts c, and log-prob. function lp:

```
numpy.exp(-numpy.mean(c*lp(t) for (t, c) in ngram_types.items()))
```

If you're computing a cross-entropy loss function (e.g., in Pytorch):

```
loss_fn = torch.nn.CrossEntropyLoss(reduction='mean')
torch.exp(loss_fn(…))
```

Example perplexity for trigram model

Trigrams	MLE p(trigram)
<BOS> <BOS> The	1
<BOS> The film	1
The film ,	0
film , a	0
, a hit	0
a hit !	0
hit ! <EOS>	0
Perplexity	???

"The film , a hit !"

$$\text{perplexity} = \exp\left(\frac{-1}{M}\sum_{i=1}^{M}\log p(w_i \mid h_i)\right)$$

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Perplexity	Infinity

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Example perplexity for trigram model

Trigrams	MLE p(trigram)	Smoothed p(trigram)
<BOS> <BOS> The	1	2/17
<BOS> The film	1	2/17
The film ,	0	1/17
film , a	0	1/16
, a hit	0	1/16
a hit !	0	1/17
hit ! <EOS>	0	1/16
Perplexity	Infinity	???

"The film , a hit !"

$$\text{perplexity} = \exp\left(\frac{-1}{M}\sum_{i=1}^{M}\log p(w_i \mid h_i)\right)$$

Example perplexity for trigram model

Trigrams	MLE p(trigram)	Smoothed p(trigram)
<BOS> <BOS> The	1	2/17
<BOS> The film	1	2/17
The film ,	0	1/17
film , a	0	1/16
, a hit	0	1/16
a hit !	0	1/17
hit ! <EOS>	0	1/16
Perplexity	Infinity	13.59

"The film , a hit !"

$$\text{perplexity} = \exp\left(\frac{-1}{M}\sum_{i=1}^{M}\log p(w_i \mid h_i)\right)$$

0s Are Not Your (Language Model's) Friend

$$p(\text{item}) \propto count(\text{item}) = 0 \rightarrow$$
$$p(\text{item}) = 0$$

0 probability → item is *impossible*

Os annihilate: x*y*z*0 = 0

Language is creative:

new words keep appearing

existing words could appear in known contexts

How much do you trust your data?

Language Models & Smoothing

~~Maximum likelihood (MLE): simple counting~~

Other count-based models

- **Laplace smoothing, add- λ** ← Easy to implement
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Neural n-gram models ← Feedforward LMs

Recurrent/autoregressive NNs ← Precursor to modern LMs

Add-λ estimation

Other names: Laplace smoothing, Lidstone smoothing

Pretend we saw each word λ more times than we did

$$p(\mathrm{z}) \propto count(\mathrm{z}) + \lambda$$

Add λ to all the counts

Add-λ estimation

Other names: Laplace smoothing, Lidstone smoothing

Pretend we saw each word λ more times than we did

Add λ to all the counts

$$p(\mathrm{z}) \propto count(\mathrm{z}) + \lambda$$
$$= \frac{count(\mathrm{z}) + \lambda}{\sum_v (count(\mathrm{v}) + \lambda)}$$

Add-λ estimation

Other names: Laplace smoothing, Lidstone smoothing

Pretend we saw each word λ more times than we did

Add λ to all the counts

$$p(\mathrm{z}) \propto count(\mathrm{z}) + \lambda$$

$$= \frac{count(\mathrm{z}) + \lambda}{W + V\lambda}$$

W: # tokens

V: # types

Add-λ N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type)	Raw Count	Norm	Prob.	Add-λ Count	Add-λ Norm.	Add-λ Prob.
The	1	16	1/16			
film	2		1/8			
got	1		1/16			
a	2		1/8			
great	1		1/16			
opening	1		1/16			
and	1		1/16			
the	1		1/16			
went	1		1/16			
on	1		1/16			
to	1		1/16			
become	1		1/16			
hit	1		1/16			
.	1		1/16			

Add-1 N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type)	Raw Count	Norm	Prob.	Add-1 Count	Add-1 Norm.	Add-1 Prob.
The	1	16	1/16	2		
film	2		1/8	3		
got	1		1/16	2		
a	2		1/8	3		
great	1		1/16	2		
opening	1		1/16	2		
and	1		1/16	2		
the	1		1/16	2		
went	1		1/16	2		
on	1		1/16	2		
to	1		1/16	2		
become	1		1/16	2		
hit	1		1/16	2		
.	1		1/16	2		

Add-1 N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type)	Raw Count	Norm	Prob.	Add-1 Count	Add-1 Norm.	Add-1 Prob.
The	1	16	1/16	2	16 + 14*1 = 30	
film	2		1/8	3		
got	1		1/16	2		
a	2		1/8	3		
great	1		1/16	2		
opening	1		1/16	2		
and	1		1/16	2		
the	1		1/16	2		
went	1		1/16	2		
on	1		1/16	2		
to	1		1/16	2		
become	1		1/16	2		
hit	1		1/16	2		
.	1		1/16	2		

Add-1 N-Grams (Unigrams)

The film got a great opening and the film went on to become a hit .

Word (Type)	Raw Count	Norm	Prob.	Add-1 Count	Add-1 Norm.	Add-1 Prob.
The	1	16	1/16	2	16 + 14*1 = 30	=1/15
film	2		1/8	3		=1/10
got	1		1/16	2		=1/15
a	2		1/8	3		=1/10
great	1		1/16	2		=1/15
opening	1		1/16	2		=1/15
and	1		1/16	2		=1/15
the	1		1/16	2		=1/15
went	1		1/16	2		=1/15
on	1		1/16	2		=1/15
to	1		1/16	2		=1/15
become	1		1/16	2		=1/15
hit	1		1/16	2		=1/15
.	1		1/16	2		=1/15

An Extended Trigram Example

The film got a great opening and the film went on to become a hit .

Q: With OOV, EOS, and BOS, how many types (for normalization)?

Context: x y	Word (Type): z	Raw Count	Add-1 count	Norm.	Probability p(z | x y)
The film	The	0			
The film	film	0			
The film	got	1			
The film	went	0			
The film	OOV	0			
The film	EOS	0			
		…			
a great	great	0			
a great	opening	1			
a great	and	0			
a great	the	0			
		…			

An Extended Trigram Example

The film got a great opening and the film went on to become a hit .

Q: With OOV, EOS, and BOS, how many types (for normalization)?

A: 16
(why don't we count BOS?)

Context: x y	Word (Type): z	Raw Count	Add-1 count	Norm.	Probability p(z | x y)
The film	The	0			
The film	film	0			
The film	got	1			
The film	went	0			
The film	OOV	0			
The film	EOS	0			
…					
a great	great	0			
a great	opening	1			
a great	and	0			
a great	the	0			
…					

An Extended Trigram Example

The film got a great opening and the film went on to become a hit .

Q: With OOV, EOS, and BOS, how many types (for normalization)?

A: 16
(why don't we count BOS?)

Context: x y	Word (Type): z	Raw Count	Add-1 count	Norm.	Probability p(z | x y)
The film	The	0	1	17 (=1+16*1)	1/17
The film	film	0	1		1/17
The film	got	1	2		2/17
The film	went	0	1		1/17
…					…
The film	OOV	0	1		1/17
The film	EOS	0	1		1/17
…					
a great	great	0	1	17	1/17
a great	opening	1	2		2/17
a great	and	0	1		1/17
a great	the	0	1		1/17
…					

An Extended Trigram Example

The film got a great opening and the film went on to become a hit .

Context: x y	Word (Type): z	Raw Count	Add-1 count	Norm.	Probability p(z | x y)
The film	The	0	1	17 (=1+16*1)	1/17
The film	film	0	1		1/17
The film	got	1	2		2/17
The film	went	0	1		1/17
…					…
The film	OOV	0	1		1/17
The film	EOS	0	1		1/17
…					
a great	great	0	1	17	1/17
a great	opening	1	2		2/17
a great	and	0	1		1/17
a great	the	0	1		1/17
…					

Q: What is the perplexity for the sentence "The film , a hit !"

What are the tri-grams for "The film , a hit !"

Trigrams	MLE p(trigram)
<BOS> <BOS> The	1
<BOS> The film	1
The film ,	0
film , a	0
, a hit	0
a hit !	0
hit ! <EOS>	0

What are the tri-grams for “The film , a hit !”

Trigrams	MLE p(trigram)	UNK-ed trigrams
<BOS> <BOS> The	1	<BOS> <BOS> The
<BOS> The film	1	<BOS> The film
The film ,	0	The film <UNK>
film , a	0	film <UNK> a
, a hit	0	<UNK> a hit
a hit !	0	a hit <UNK>
hit ! <EOS>	0	hit <UNK> <EOS>

What are the tri-grams for "The film , a hit !"

Trigrams	MLE p(trigram)	UNK-ed trigrams	Smoothed p(trigram)
<BOS> <BOS> The	1	<BOS> <BOS> The	2/17
<BOS> The film	1	<BOS> The film	2/17
The film ,	0	The film <UNK>	1/17
film , a	0	film <UNK> a	1/16
, a hit	0	<UNK> a hit	1/16
a hit !	0	a hit <UNK>	1/17
hit ! <EOS>	0	hit <UNK> <EOS>	1/16

Setting Hyperparameters

Use a **development** corpus

Training Data	Dev Data	Test Data

Choose λs to maximize the probability of dev data:

- Fix the N-gram probabilities (on the training data)
- Then search for λs that give largest probability to held-out set:

Language Models & Smoothing

~~Maximum likelihood (MLE): simple counting~~

~~Other count-based models~~

- ~~Laplace smoothing, add-λ~~
- ~~Interpolation models~~
- ~~Discounted backoff~~
- ~~Interpolated (modified) Kneser-Ney~~
- ~~Good-Turing~~
- ~~Witten-Bell~~

Easy to implement

Advanced/ out of scope

Maxent n-gram models ← Featureful LMs

Neural n-gram models ← Feedforward LMs

Recurrent/autoregressive NNs ← Precursor to modern LMs

Maxent Models as Featureful n-gram Language Models

p(Colorless green ideas sleep furiously | Label) =
p(Colorless | Label, <BOS>) * … * p(<EOS> | Label , furiously)

Model each n-gram term with a maxent model

$$p(x_i \mid y, x_{i-N+1:i-1}) = \text{maxent}(y, x_{i-N+1:i-1}, x_i)$$

generatively trained:
learn to model *(class-specific) language*

Language Model with Maxent n-grams

$$p_n(\text{🗎} \mid y) = \prod_{i=1}^{M} \text{maxent}(\overset{\text{label}}{y}, \underbrace{x_{i-n+1:i-1}, x_i}_{\text{n-gram}})$$

$$= \prod_{i=1}^{M} \frac{\exp(\theta_{x_i}^T f(y, x_{i-n+1:i-1}))}{\sum_{x'} \exp(\theta_{x'}{}^T f(y, x_{i-n+1:i-1}))}$$

Iterate through all possible output vocab types x'---just like in count-based LMs

What Should These Features Do?

$p(x_i \mid y, x_{i-N+1:i-1}) = \text{maxent}(y, x_{i-N+1:i-1}, x_i)$, e.g.,

$$p(\text{sleep} \mid y, \text{green}, \text{ideas}) = \\ \text{maxent}\big(y, x_{i-2,i-1} = (\text{green}, \text{ideas}), x_i = \text{sleep}\big) \\ \propto \exp(\theta_{x_i=\text{sleep}}{}^T f(y, x_{i-2,i-1} = (\text{green}, \text{ideas})))$$

(in-class discussion)

N-gram Language Models

given some context…

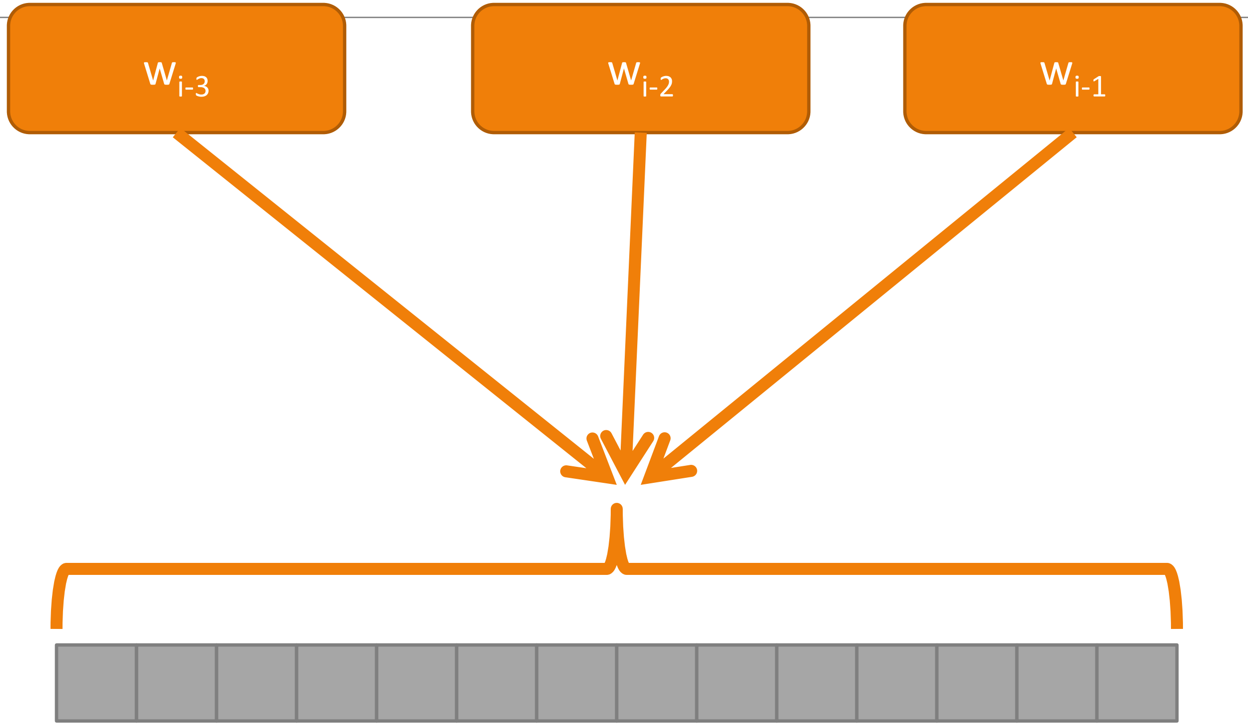
w_{i-3} w_{i-2} w_{i-1}

predict the next word

w_i

N-gram Language Models

given some context…



compute beliefs about what is likely…

$$p(w_i |\, w_{i-3}, w_{i-2}, w_{i-1}) \propto count(w_{i-3}, w_{i-2}, w_{i-1}, w_i)$$

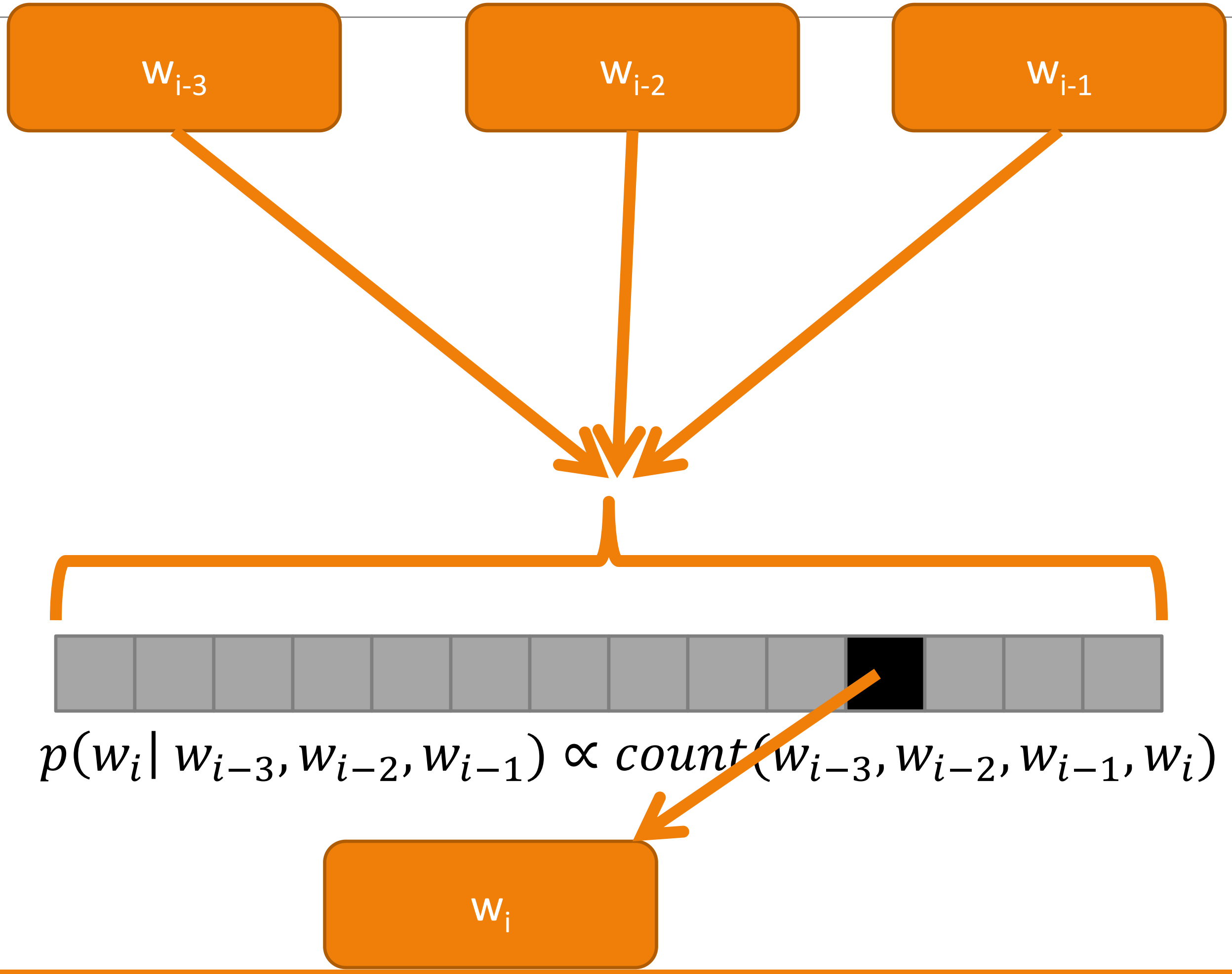
predict the next word

N-gram Language Models

given some context…

compute beliefs about what is likely…

predict the next word



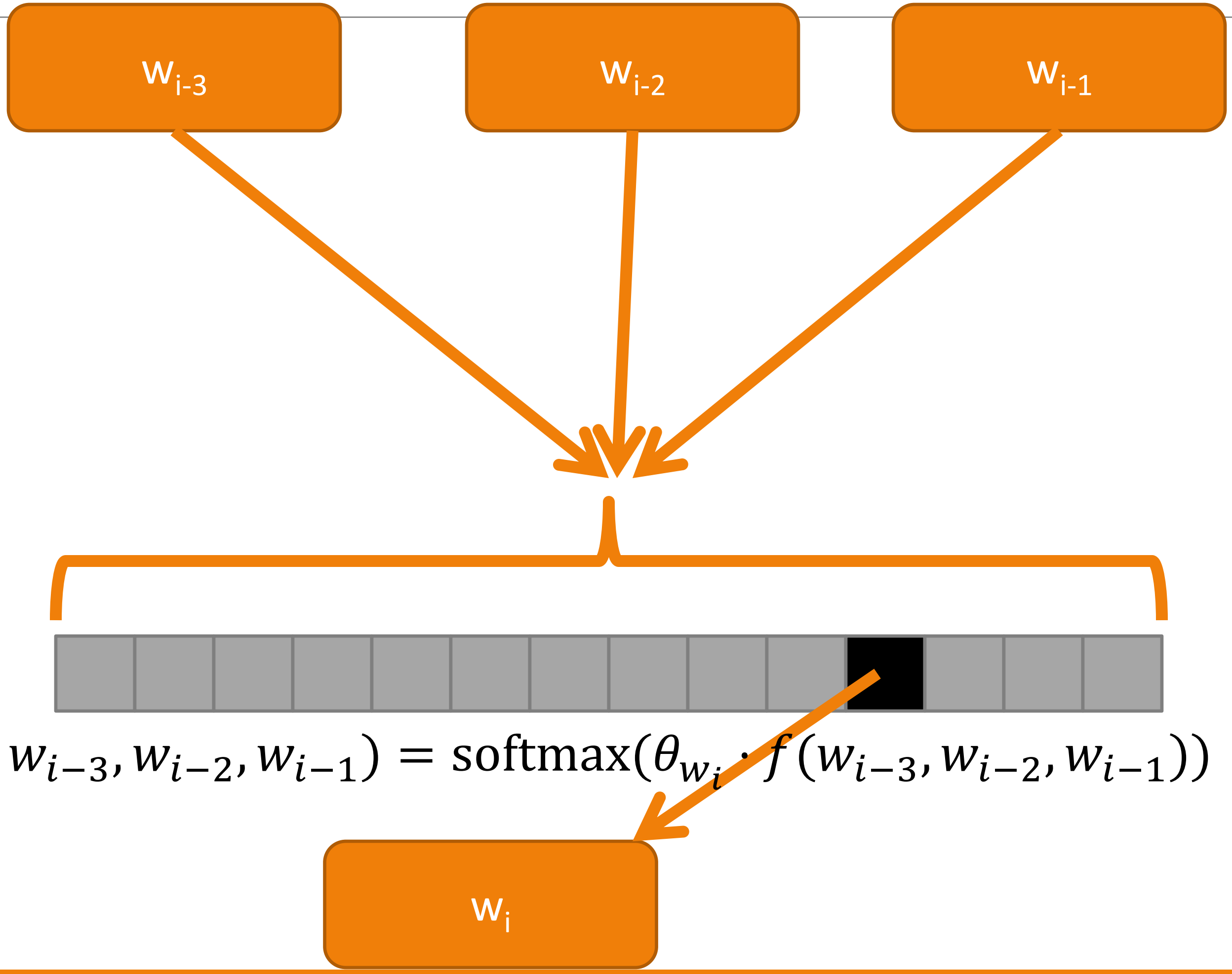
$$p(w_i \mid w_{i-3}, w_{i-2}, w_{i-1}) \propto count(w_{i-3}, w_{i-2}, w_{i-1}, w_i)$$

Maxent Language Models

given some context…

compute beliefs about what is likely…

predict the next word



$$p(w_i \mid w_{i-3}, w_{i-2}, w_{i-1}) = \text{softmax}(\theta_{w_i} \cdot f(w_{i-3}, w_{i-2}, w_{i-1}))$$

A Closer Look at Maxent $p($ Won’t you please donate? $|$ Primary $)$

This is a *class-based* language model, but incorporate the label into the features

To learn $p($ Won’t you please donate? $|$ Class $)$:

Define features f that make use of the specific label Class

Unlike count-based models, you don’t *need* “separate” models here