PERCEPTRONS

Lara J. Martin (she/they)
TA: Aydin Ayanzadeh (he)

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CMSC 671

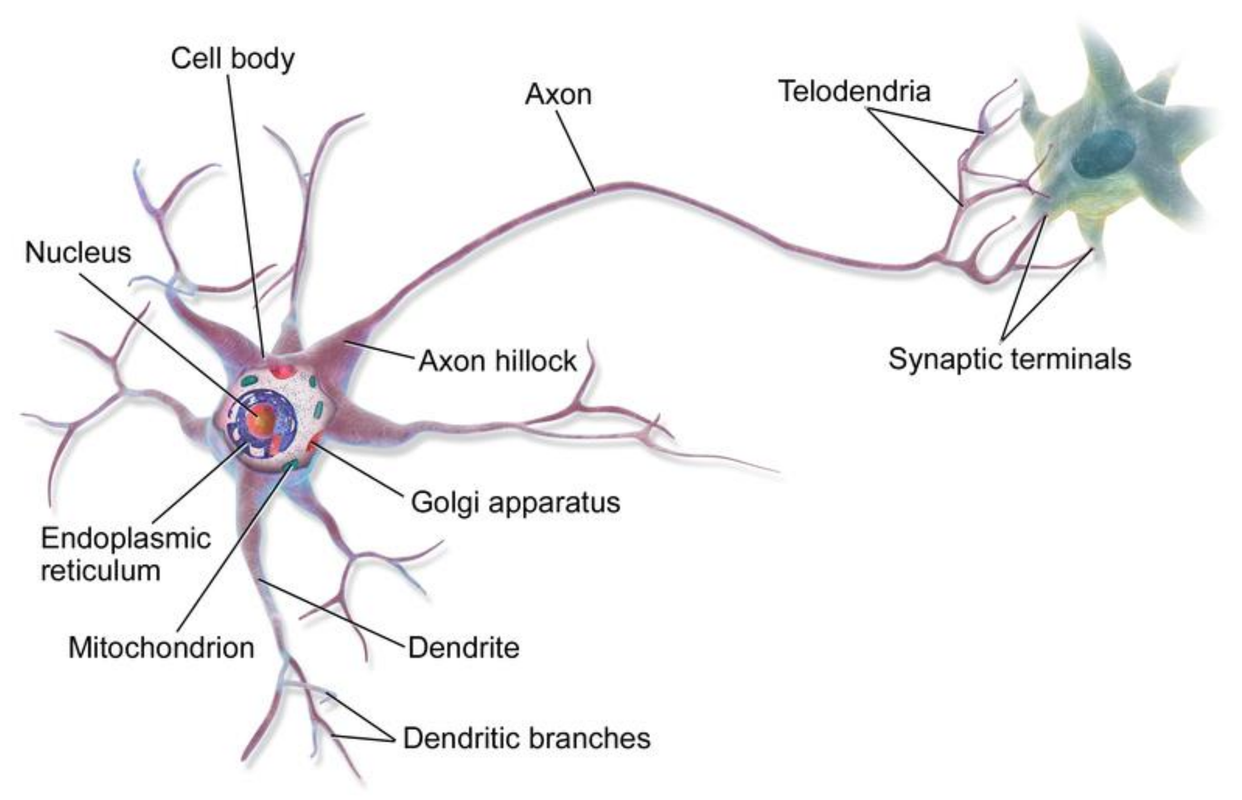
By the end of class today, you will be able to:

1. Compute the inputs and outputs for individual neurons
2. Identify the limitations of what a single-layer perceptron can represent

Slides by Dr. Cassandra Kent

BIOLOGICALLY-INSPIRED LEARNING MODELS

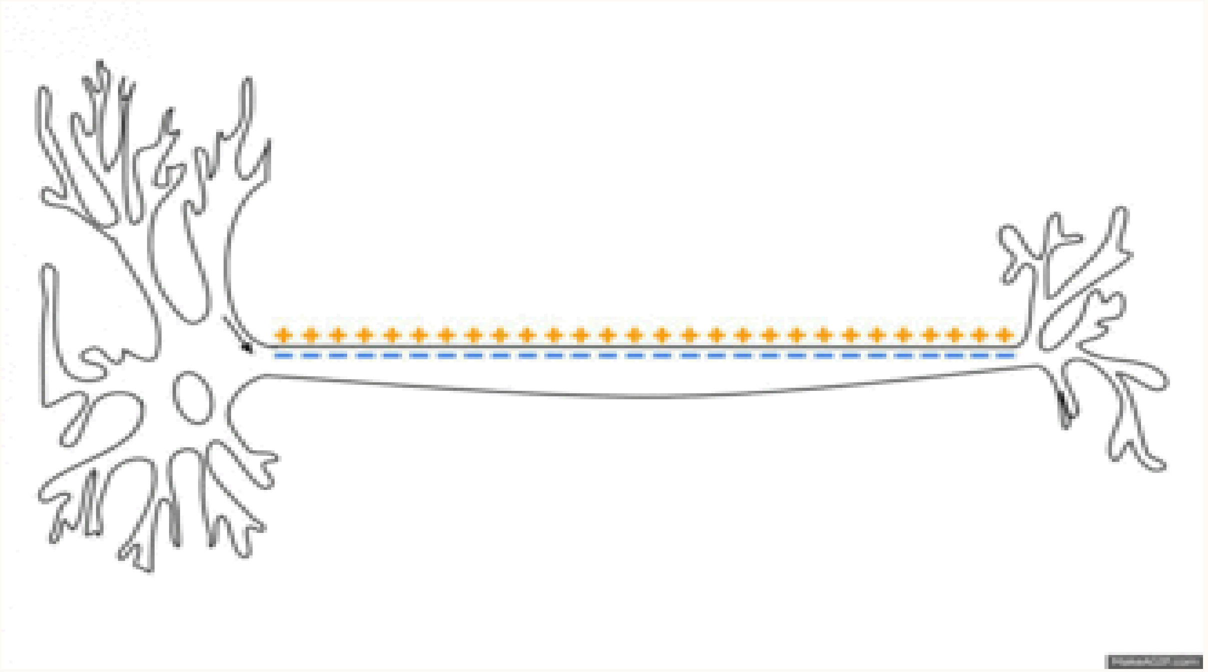
- Our mental activity consists mainly through networks of brain cells, or *neurons*
- A neuron receives a signal from one or more other neurons through *synapses*
- *Dendrites* take information, *axons* give information



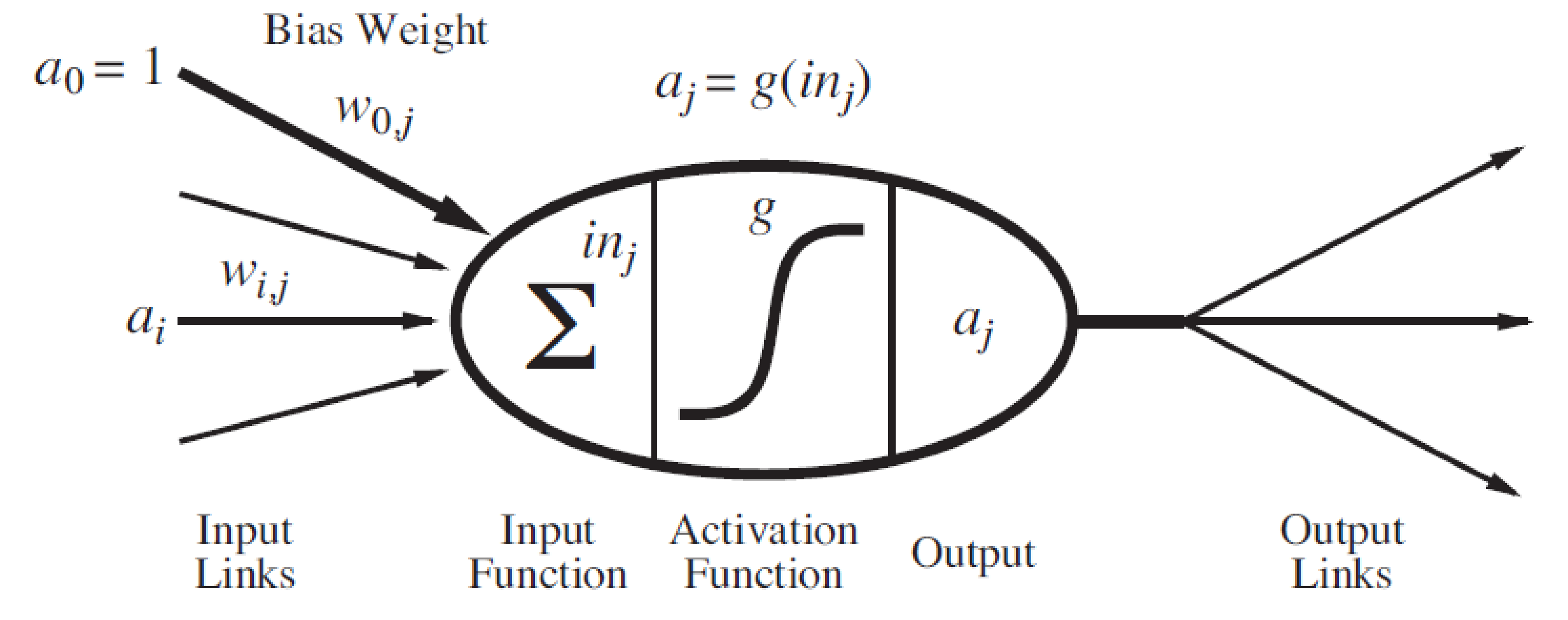
BIOLOGICALLY-INSPIRED LEARNING MODELS

Action potential (simplified):

- Each signal has a weight (w) associated with it, which can be positive or negative
- A neuron fires if the sum of the weights exceeds a threshold value
- When a neuron fires, it propagates the signal through its axon to all neurons with dendrites connected to that axon



BIOLOGICALLY-INSPIRED LEARNING MODELS: NEURON UNIT

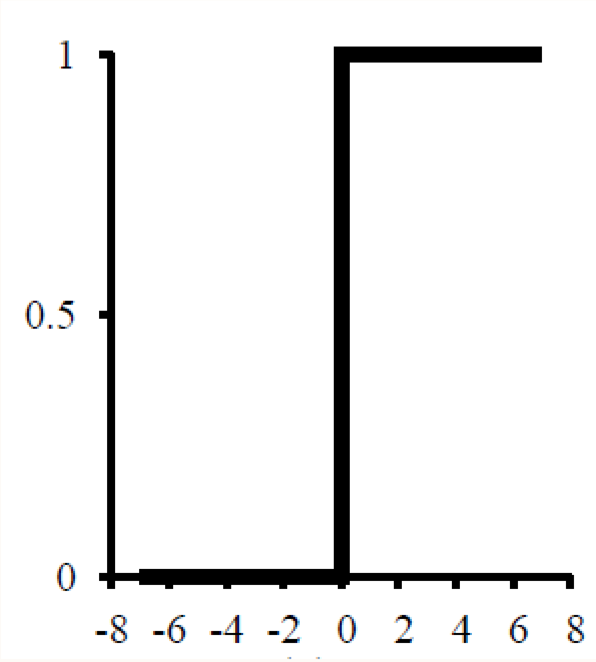


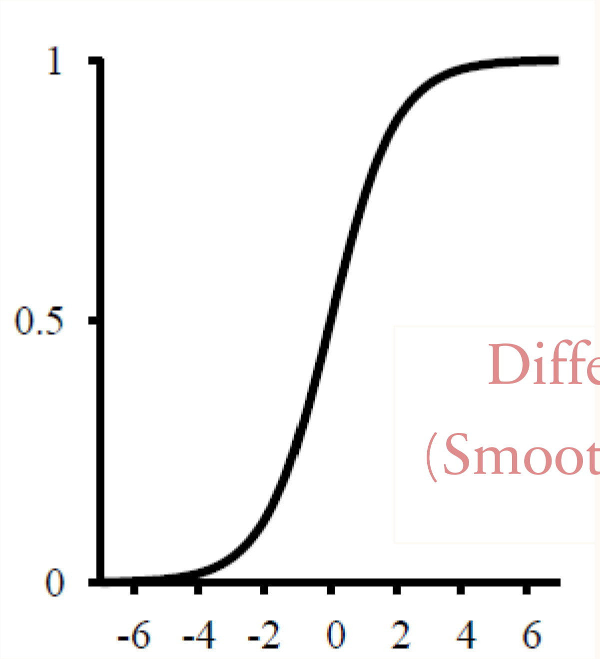
$$in_j = w_{0j} + w_{1j}a_1 + w_{2j}a_2 + \cdots + w_{ij}a_i$$

activations
$0 \leq a_i \leq 1$

weights
$-\infty < w_{ij} < \infty$

NEURON UNIT ACTIVATION FUNCTIONS





Differentiable!
(Smooth derivative)

Step function (hard threshold):

$$g(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Sigmoid function (soft threshold):

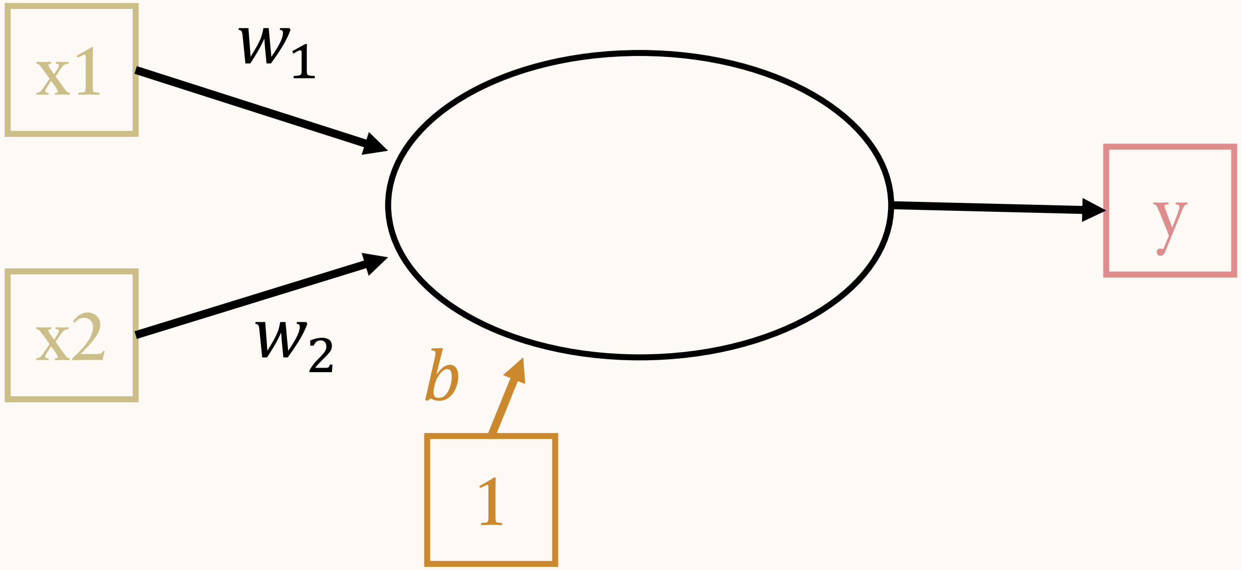
$$g(x) = \frac{1}{1 + e^{-x}}$$

INDIVIDUAL NEURON UNIT EXAMPLE

- Neurons can represent different functions, depending on their weights.
- Can we implement the binary AND function?

Binary AND
Inputs and outputs

	1	0
1	1	0
0	0	0



INDIVIDUAL NEURON UNIT EXAMPLE

What weights implement AND?

$$w_1 = 1;\ w_2 = 1; w_0 = ???$$

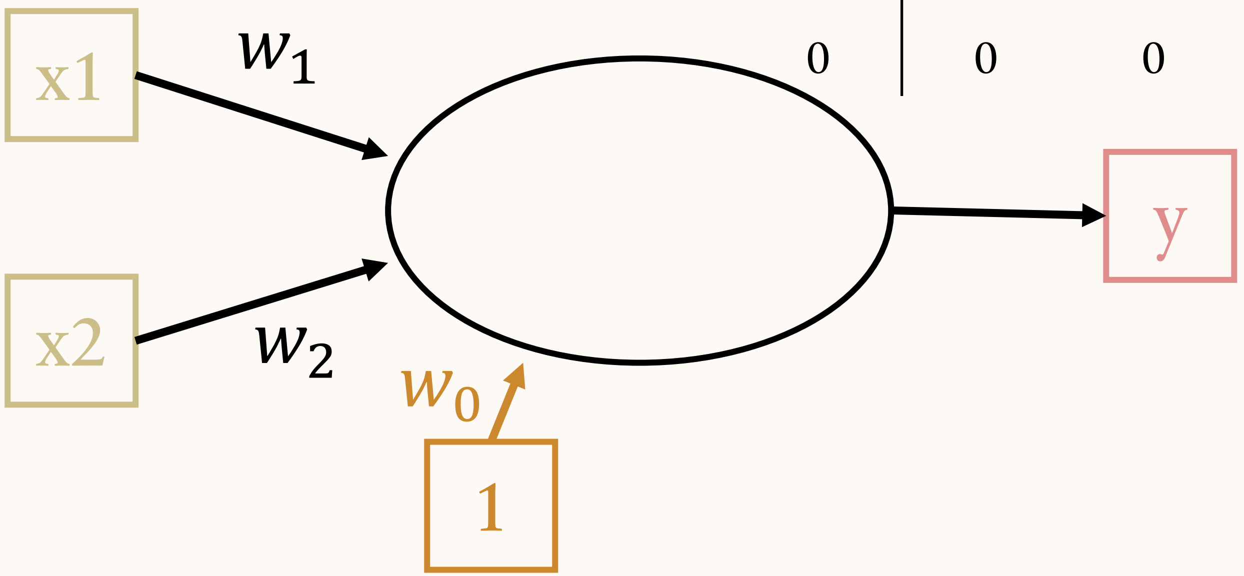
$$f(1,1) = g(1*1 + 1*1 + w_0) = g(2 + w_0) = 1$$

$$f(1,0) = g(1*1 + 1*0 + w_0) = g(1 + w_0) = 0$$

$$f(0,0) = g(1*0 + 1*0 + w_0) = g(w_0) = 0$$

Binary AND
Inputs and outputs

	1	0
1	1	0
0	0	0



INDIVIDUAL NEURON UNIT EXAMPLE

What weights implement AND?

$$w_1 = 1;\ w_2 = 1; w_0 = -1.5$$

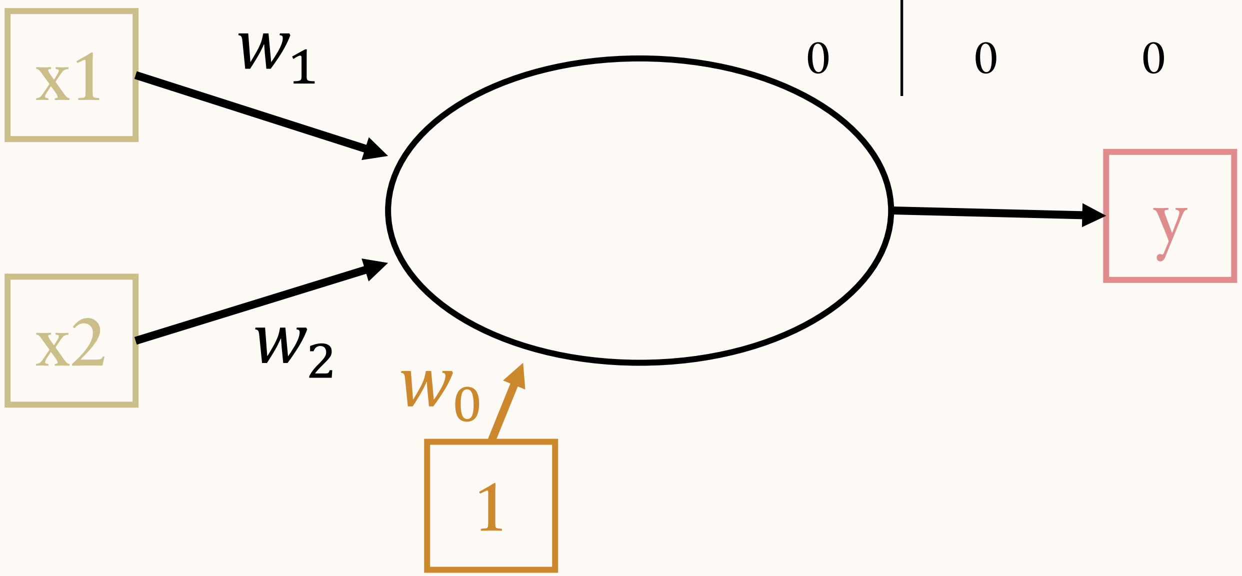
$$f(1,1) = g(1*1+1*1+w_0) = g(2+w_0) = 1$$

$$f(1,0) = g(1*1+1*0+w_0) = g(1+w_0) = 0$$

$$f(0,0) = g(1*0+1*0+w_0) = g(w_0) = 0$$

Binary AND
Inputs and outputs

	1	0
1	1	0
0	0	0

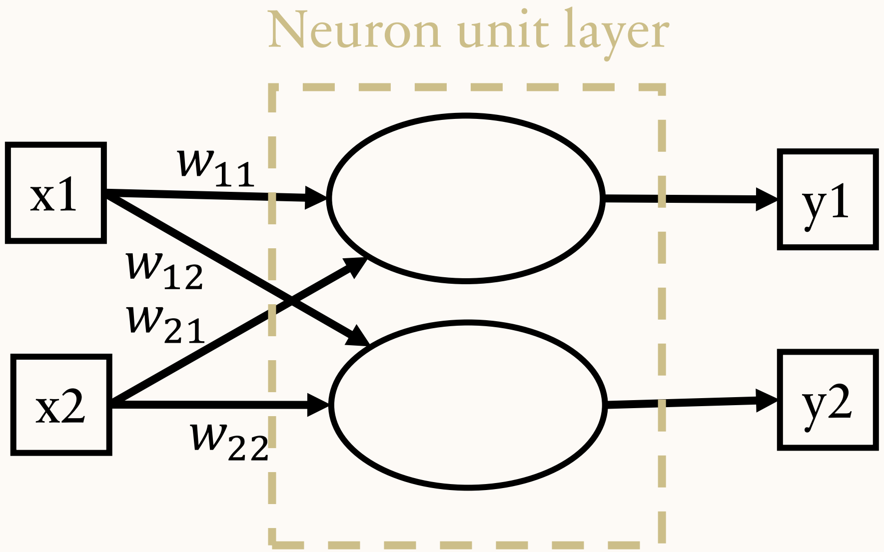


CONSTRUCTING COMPLEX FUNCTIONS AS NETWORKS

We represent complex functions by combining neurons in networks.

Perceptron: single-layer feed-forward neural network

(connections go in one direction, DAG)



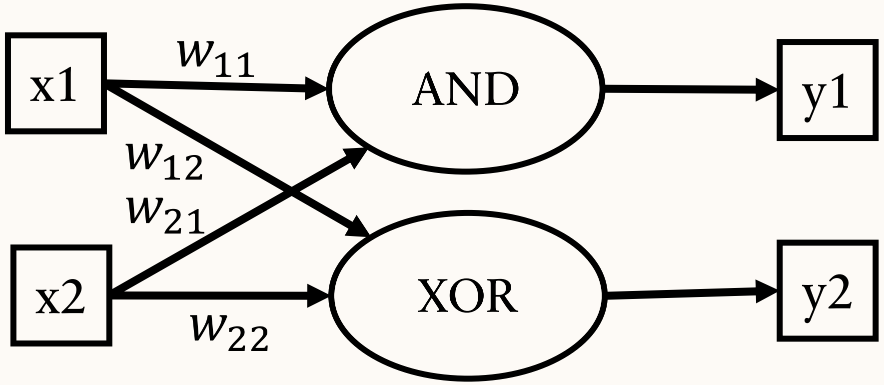
CONSTRUCTING COMPLEX FUNCTIONS AS NETWORKS

We represent complex functions by combining neurons in networks.

Perceptron: single-layer feed-forward neural network

Two-bit addition function inputs/outputs

x1	x2	y1 (carry)	y2 (sum)
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0



INDIVIDUAL NEURON UNIT PRACTICE

Let's try implementing other binary logic functions as neurons.

Determine weights that implement these functions:

Binary NOT
Inputs and outputs

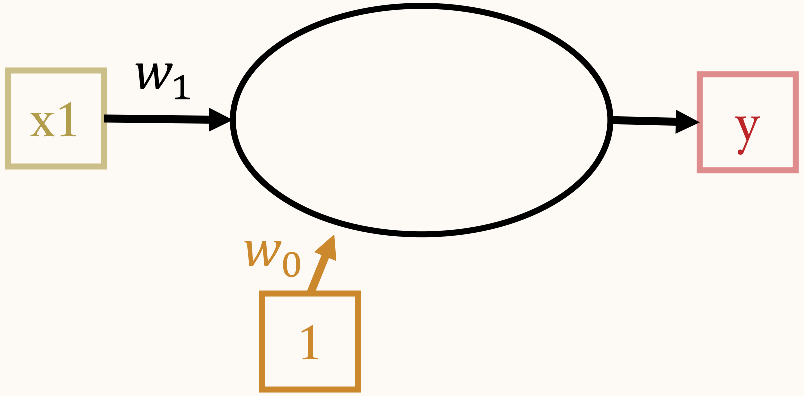
x	y
1	0
0	1

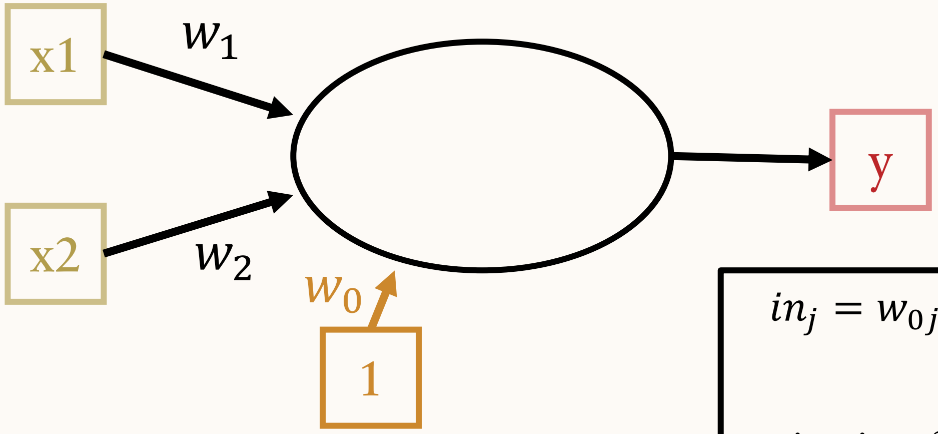
Binary OR
Inputs and outputs

x1, x2	1	0
1	1	1
0	1	0

Binary XOR
Inputs and outputs

x1, x2	1	0
1	0	1
0	1	0





$$in_j = w_{0j} + w_{1j}a_1 + w_{2j}a_2 + \cdots + w_{ij}a_i$$

$$-\infty < w_{ij} < \infty$$

activation function: $g(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$

INDIVIDUAL NEURON UNIT PRACTICE

Binary NOT
Inputs and outputs

x	y
1	0
0	1

Binary OR
Inputs and outputs

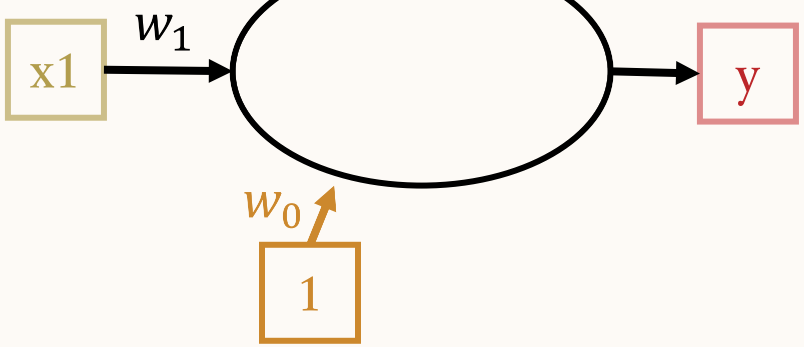
x1, x2	1	0
1	1	1
0	1	0

Binary XOR
Inputs and outputs

x1, x2	1	0
1	0	1
0	1	0

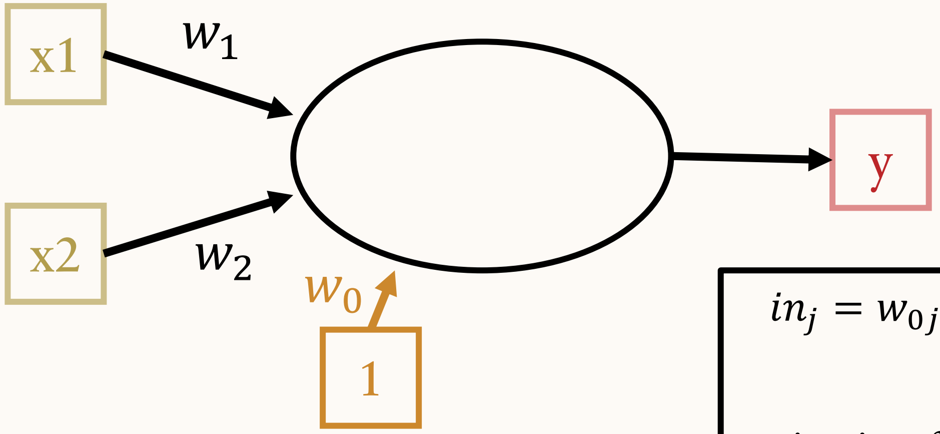
What weights implement **NOT**?

$$w_1 = -1;\ b = 1$$



What weights implement **OR?**

$$w_1 = 1;\ w_2 = 1; b = -0.5$$



$$in_j = w_{0j} + w_{1j}a_1 + w_{2j}a_2 + \cdots + w_{ij}a_i$$
$$-\infty < w_{ij} < \infty$$
$$\text{activation function: } g(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

WHAT CAN WE REPRESENT WITH A PERCEPTRON?

- Each single unit represents a linear function (a linear combination of the inputs) and a non-linear activation function
- As a result, perceptrons represent binary classifiers for linearly-separable data

Binary NOT
Inputs and outputs

x	y
1	0
0	1

Binary AND
Inputs and outputs

x1, x2	1	0
1	1	0
0	0	0

Binary OR
Inputs and outputs

x1, x2	1	0
1	1	1
0	1	0

Binary XOR
Inputs and outputs

x1, x2	1	0
1	0	1
0	1	0

?

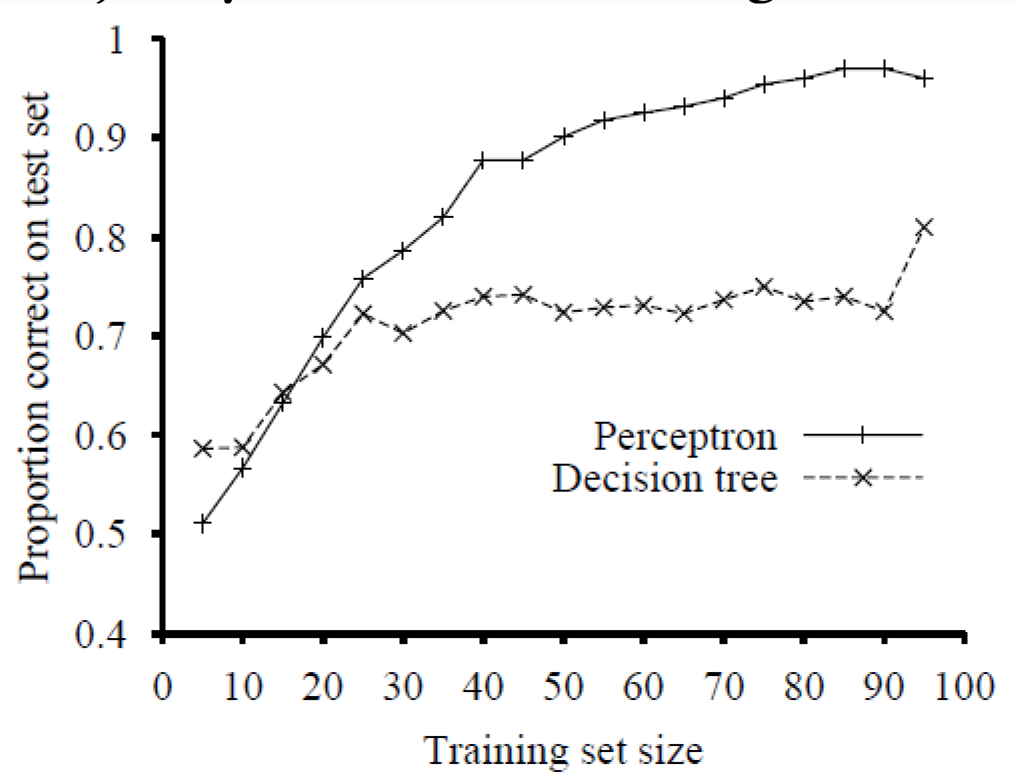
PERCEPTRON LIMITATIONS: COMPLEX EXAMPLE

- Perceptrons **can** effectively represent linearly-separable functions
- Example: **majority function**: output 1 only if more than half of the inputs are 1

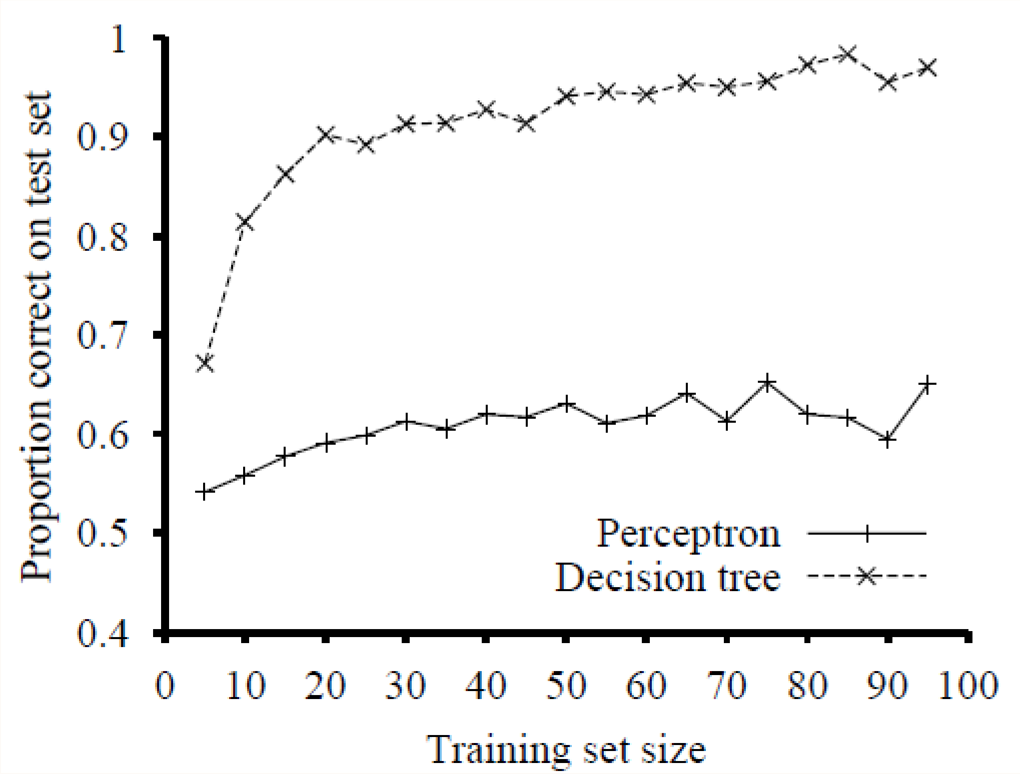
- Perceptrons **can not** effectively represent complex *non-linearly-separable* functions
- Example: **restaurant problem**: given a set of features for a restaurant, will we wait for a table?

PERCEPTRON LIMITATIONS: COMPLEX EXAMPLE

Majority function learning curves



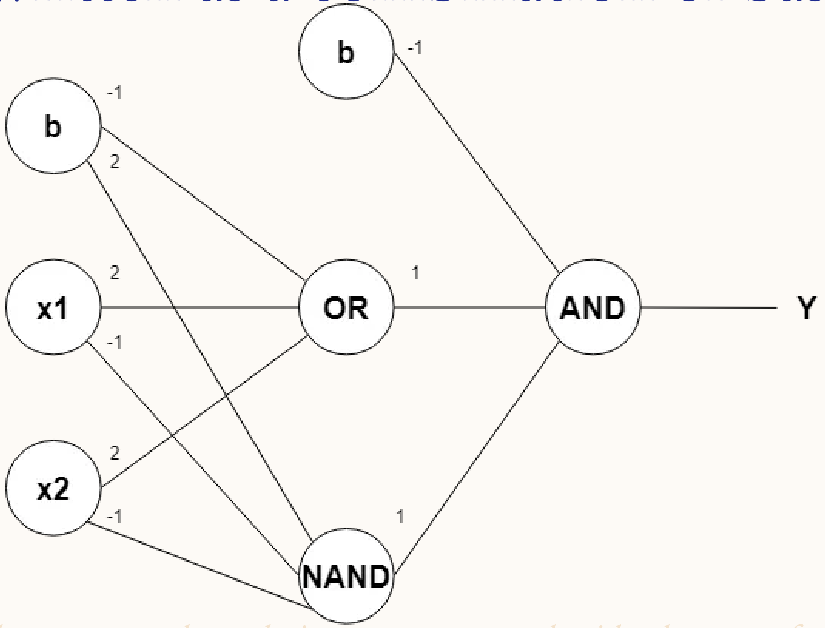
Restaurant problem learning curves



MULTI-LAYER FEED-FORWARD NEURAL NETWORKS

- Single-layer perceptrons can only classify linearly separable data, but…
- **Multi-layer** neural networks can overcome this limitation!

- Example: XOR can be written as a combination of basic logic functions



https://medium.com/@stanleydukor/neural-representation-of-and-or-not-xor-and-xnor-logic-gates-perceptron-algorithm-b0275375fea1